SNAP Centre Workshop

Standard Order of Operations

Standard Order of Operations

The importance of using a standardized order of operations can be seen in most mathematic expressions that involve more than one operation.

Example 1 $7 - 3 \times 2$ $Evaluate.$

First, evaluate the expression by performing each operation from left to right.

$$7 - 3 \times 2$$

$$= 4 \times 2$$

$$= \mathbf{8}$$

Next, evaluate the expression by performing multiplication first, followed by subtraction.

$$7 - 3 \times 2$$

$$= 7 - 6$$

$$= \mathbf{1}$$

The exact same operations were performed. Changing the order in which the operations were performed, however, changed the final answer.

To avoid the potential confusion illustrated in **Example 1**, a standardized order of operations was introduced.

1) Grouped operations (usually in **parentheses** or **brackets**) are evaluated**.**
2) **Exponents** are evaluated.
3) **Multiplication** and **division** operations are performed, left-to-right.
4) **Addition** and **subtraction** operations are performed, left-to-right.

This order indicates that any operations contained within parentheses (also called "brackets") are performed before all others. Next, all terms raised to an exponent are evaluated. After all exponents have been evaluated, multiplication and division operations are performed. Finally, after all other operations have been completed, addition and subtraction operations are performed.

Note that there is no distinction made between multiplication and division, or addition and subtraction. This is because division can be considered multiplication by a reciprocal (e.g. Dividing by 4 is the same as multiplying by ¼) and subtraction can be considered addition of a negative number (e.g. subtracting 1 is the same as adding -1).

Example 2 3×4^2 $Evaluate.$

*This problem involves an exponential term, as well as multiplication. Even though "3 * 4" is the first operation when reading the equation left-to-right, referring back to the order of operations clearly indicates that the exponent should be evaluated first, after which the multiplication can be performed.*

3×4^2

$= \ 3 \times 16$

$= \ \mathbf{48}$

Example 3 $\quad 32 \ \div \ (2 \times \ 2)^2 \times \ 2 \qquad Evaluate.$

Since 2×2 *is contained in parentheses, this operation is performed first.*

$32 \ \div \ (2 \times 2)^2 \times \ 2$

$= 32 \div 4^2 \times 2$

Next, before performing multiplication or division, the term being raised to an exponent is evaluated.

$= \ 32 \ \div \ 16 \times 2$

Multiplication and division are the same with regards to their place in the order of operations, so they are performed left-to-right when presented together. In this case, that means division is performed first, followed by multiplication.

$= \ 2 \times 2$

$= \ \mathbf{4}$

Nested Parentheses

Example 4 $\quad \left((2(3^2 \ \div \ 9) + \ 7) \div \ 3\right)^2 \qquad Evaluate.$

*This example contains **nested parentheses**, where there are terms grouped in parentheses inside another set of parentheses.*

Nested parentheses are evaluated by starting with the operations in the inner-most set of parentheses, and working to the outer-most set, while making sure to follow the correct order of operations inside each set of parentheses.

$= \ \left((2(9 \ \div \ 9) + \ 7) \div \ 3\right)^2$

$= \ \left((2(1) + \ 7) \div \ 3\right)^2$

$= \ \left((2 \ + \ 7) \div \ 3\right)^2$

The inner-most set of parentheses featured an exponent and division. Following the standard order, the exponent was evaluated first, after which the division was performed. This resolved the inner-most set of parentheses. We will continue this process until there are no more terms grouped in parentheses and the entire expression has been evaluated.

$= \ \left((9) \div \ 3\right)^2$

$$= (3)^2$$

$$= \mathbf{9}$$

Stacked Exponents

When presented with stacked exponents, where the term in the exponent is also raised to an exponent, the "top" term is evaluated first.

Example 5 $2^{1^{5^2}}$ $Evaluate.$

We begin by evaluating our top term, which is 5^2*.*

$$= 2^{1^{25}}$$

This leaves us with a new top term to evaluate, 1^{25}

$$= 2^1$$

$$= \mathbf{2}$$

Notice that is we had treated our base 2 *as our innermost term, our final answer would have been* 1024*.*

Fractions

Fractions are another way of grouping operations, similar to parentheses.

Example 6 $\frac{9-21 \div 7}{3*(4-3)}$ $Evaluate.$

All terms in the numerator are considered a group, and so are the terms in the denominator. As such, the operations in these groups must be completed before the fraction can be reduced.

$$= \frac{9-3}{3*(1)}$$

$$= \frac{6}{3}$$

$$= \mathbf{2}$$

With the operations in the numerator and denominator performed, the fraction can be reduced. In this case, it reduces to a whole number.

Order of Operations Involving Absolute Values

Example 7 Evaluate the following expression:

$$\frac{|7-20| \times 3 - |2 \times (-3)|^2}{9}$$

When dealing with terms inside absolute value bars, treat them as if they are a group inside parentheses. This means that even though multiplication takes precedence over subtraction, $7-20$ *is performed before* 20×3*, and* $2\times(-3)$ *is performed before* $(-3)^2$ *despite exponents taking precedence over multiplication.*

$$= \frac{|-13|\times 3 - |-6|^2}{9}$$

$$= \frac{13\times 3 - 6^2}{9}$$

$$= \frac{39-36}{9}$$

$$= \frac{3}{9}$$

After all operations in the numerator are completed, the fraction can be reduced.

$$= \mathbf{\frac{1}{3}}$$

Functions

When dealing with expressions that contain functions such as logarithmic functions or trigonometric functions, we treat the function's argument or input the same way we would a set of parentheses.

Example 8 $\quad 2\sin^2\left(2\left(\frac{\pi}{12}+\frac{\pi}{12}\right)\right) \quad$ $Evaluate.$

First, we evaluate the addition in our innermost parentheses, then simplify.

$$= 2\sin^2\left(2\left(\frac{2\pi}{12}\right)\right)$$

$$= 2sin^2\left(\frac{4\pi}{12}\right)$$

$$= 2\sin^2\left(\frac{\pi}{3}\right)$$

Next, we evaluate the value given by the sine function. Make sure to note that the function is being squared.

$$= 2\left(\sin\frac{\pi}{3}\right)^2$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2\left(\frac{3}{4}\right)$$

$$= \frac{6}{4}$$

$$= \mathbf{\frac{3}{2}}$$